and 7.6×10^4 newtons/m^2 (0.75 atm) when the accelerator is operating.

9. Future Accelerators

The present accelerator, although its operation is limited in duration to about 3 sec, serves well during this phase of the research program. To overcome the disadvantage of such short running time, a water-cooled version of it has been designed and constructed.

A larger accelerator has also been designed to give an exit velocity of 12,000 m/sec at a density of 3×10^{-4} kg/m^3. The accelerator will have a channel of 2.5- $\times$ 2.5-in. cross section and will be 18 in. long. It will be operated from a 10-Mw d.c. supply, and a second 10-Mw d.c. supply will power the arc heater.

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Energy Loss in Pulsed Coaxial Plasma Guns

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The one-dimensional flow of a current-carrying plasma, driven into neutral gas by its self-magnetic field, is considered for the case where both the neutral gas and the current have arbitrary distributions in the direction of flow. The conservation laws are used to derive a simple expression for the amount of input power that goes into all forms of internal energy. It is shown that this power depends mainly on the rate at which the current sheet sweeps up neutral gas. In principle, internal energy can be converted into directed energy by expansion; however, it is argued that, in the coaxial plasma accelerators currently used in propulsion studies, most of the internal energy will be lost by excitation and subsequent radiation or ionization before expansion occurs.

I. Introduction

Two simple models are commonly used to describe the operation of coaxial guns.[1] The "slug model"[2] assumes that a plasma of constant mass is accelerated into a vacuum, whereas the "shock model"[2,3] assumes that the barrels are uniformly filled with neutral gas that is ionized and accelerated by a shock driven by the $j \times B$ forces. The conservation laws show that, when a shock moves with constant velocity into a stationary gas, half of the energy supplied to the plasma appears in directed motion, whereas the other half is taken by internal energy of one form or another; the slug model, however, avoids this energy loss. In a real accelerator, the gas distribution will lie between the extremes described by the two models.

This paper has two objectives; firstly to obtain a general expression for the energy loss that occurs when the gas distribution ahead of the current sheet is arbitrary and secondly to consider the implications of this expression for the coaxial guns currently used to investigate the problems of pulsed plasma propulsion.

II. Theory†

A one-dimensional model with plane electrodes is considered (Fig. 1). It is assumed that all quantities vary only in the z direction and with time. The main variables are

$\mathbf{B} = [0, B(z), 0]$ = magnetic flux density
$\mathbf{u} = [0, 0, u(z)]$ = fluid velocity
$p(z)$ = fluid pressure
$\rho(z)$ = fluid density

Two planes in the xy plane are defined. The first plane at $z = z_1$ moves with velocity v_1 and marks the boundary between magnetic field plus plasma and magnetic field alone; it will be referred to as the piston. Because no plasma flows through this plane, $u_1 = v_1$. The second plane at $z = z_2$ moves with velocity v_2 and marks the boundary between the initial stationary gas and gas that has been affected by the moving piston. Electric currents and plasma motion are restricted to the region between planes 1 and 2.

The conservation laws may be written as follows:

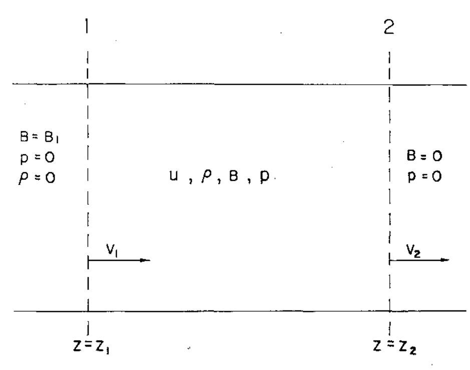

Fig. 1 Diagram illustrating the theoretical model

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† Based on an analysis in Ref. 4.

$$\frac{d}{dt}\int_1^2 \rho dz = \frac{dM_2}{dt} \tag{1}$$

$$\frac{B_1^2}{2\mu_0} = \frac{d}{dt}\int_1^2 \rho u dz = \frac{d}{dt}\int_1^2 u dM \tag{2}$$

$$\frac{B_1^2}{2\mu_0} v_1 = \frac{d}{dt}\int_1^2 \left(\frac{B^2}{2\mu_0} + \frac{\rho u^2}{2} + e + \frac{p}{\gamma - 1}\right) dz$$
$$= \frac{d}{dt}\int_1^2 \left(\frac{B^2}{2\mu_0 \rho} + \frac{u^2}{2} + \frac{e}{\rho} + \frac{p}{(\gamma - 1)\rho}\right) dM \tag{3}$$

where e = energy of excitation and ionization. Radiation losses can be taken into account by considering this excitation energy to include the energy of states that have decayed and radiated their energy out of the system. Equation (2) assumes that $B = 0$ and $p = 0$ at z_2, whereas Eq. (3) assumes that the magnetic flux is not diffusing past the piston (i.e., no resistive potential drop at the piston). This last assumption does not preclude the possibility that magnetic flux is trapped between planes 1 and 2 at the start of the discharge. Within the assumptions outlined, the three equations are general and apply to arbitrary one-dimensional distributions of B, ρ, p, and u.

From Eqs. (2) and (3)

$$\frac{dW_i}{dt} = \frac{d}{dt}\int_1^2 \left(\frac{B^2}{2\mu_0 \rho} + \frac{e}{\rho} + \frac{p}{\rho(\gamma - 1)}\right) dM =$$
$$v_1 \frac{d}{dt}\int_1^2 u dM - \frac{d}{dt}\int_1^2 \frac{u^2}{2} dM \tag{4}$$

where W_i is the total energy that is not in directed motion. Now let

$$\int_1^2 u dM = M_2 \bar{u} \tag{5}$$

where $\bar{u}$ is the average velocity of the moving fluid. The minimum energy necessary to produce momentum $M_2\bar{u}$, is $\frac{1}{2} M_2\bar{u}^2$, and any spread in fluid velocity around $\bar{u}$ involves wasted energy as far as momentum production for propulsion is concerned; if this wasted energy is W_v, then by definition

$$W_v = \int_1^2 \frac{u^2}{2} dM - \frac{1}{2} M_2 \bar{u}^2 \tag{6}$$

From Eqs. (4–6)

$$\frac{d}{dt}(W_i + W_v) = v_1 \frac{d}{dt}(M_2\bar{u}) - \frac{1}{2}\frac{d}{dt}(M_2\bar{u}^2) \tag{7}$$

Let

$$v_1 = \bar{u} + \epsilon \tag{8}$$

then from Eqs. (7) and (8)

$$\frac{d}{dt}(W_i + W_v) = \frac{1}{2}\frac{dM_2}{dt}\bar{u}^2 + \epsilon \frac{d}{dt}(M_2\bar{u}) \tag{9}$$

From Eqs. (9) and (2)

$$\frac{d}{dt}(W_i + W_v) = \frac{1}{2}\frac{dM_2}{dt}\bar{u}^2 + \epsilon \frac{B_1^2}{2\mu_0} \tag{10}$$

Equation (10) gives the rate of energy flow into processes that do not add momentum to the exhaust. The physical significance of this simple equation can be seen by transforming to a coordinate system moving with the average velocity of the moving fluid $\bar{u}$. Gas, which previously was stationary, now streams into the region between z_1 and z_2 and brings in energy at the rate $(\bar{u}^2/2)\, dM_2/dt$, whereas the magnetic pressure, acting at the piston, does work at the rate $\epsilon B_1^2/2\mu_0$. The sign of this last term depends on whether the plasma is being compressed or is expanding. If $d(W_i + W_v)/dt$ is negative, then the energy $(W_i + W_v)$ is converted to a form that does contribute momentum to the exhaust; this situation can occur only when the plasma expands, so that $\epsilon B_1^2/2\mu_0$ is also negative.

In general, the total energy lost is given by integrating Eq. (10). This integration involves the detailed fluid behavior that only can be determined by solving the problem in full. However, if the neutral gas is swept into a dense sheet so that $u = v_1 = v_2$, as in the snow-plow model, then Eq. (10) can be integrated to give

$$W_i + W_v = \frac{1}{2}\int_0^t \rho v_1^3 dt \tag{11}$$

The energy loss given by Eq. (11) can be determined from measurements of the current sheet speed and the initial gas distribution; this equation will be a good approximation of the loss incurred by a moving current sheet, unless a substantial amount of energy is in particle pressure and magnetic pressure that can be reclaimed by expansion. In the following section, the amount of energy in these two forms is estimated and compared with the amount of energy in directed motion; it will be shown that in typical pulsed coaxial guns expansion is unlikely to release an appreciable amount of energy.

III. Estimate of the Energy Reclaimed by Expansion

The magnetic energy stored in the current sheet is a small fraction of the energy supplied to the accelerator unless the sheet thickness is comparable with the distance it has moved. Normally this only occurs at early times when currents start to flow (e.g., see Ref. 5). For most of the acceleration period, the amount of magnetic energy that can be reclaimed by expansion of the current sheet is small as compared with the energy supplied to the accelerator.

The possibility of reclaiming the energy in electron thermal motion is considered next. For efficient propulsion, the energy in directed ion motion must be about 50 ev, or the "frozen flow losses" due to ionization will be excessive; this requirement favors material of high molecular weight. If a substantial amount of electron thermal energy is to be reclaimed by expansion, then the electron temperature T_e also must be about 50 ev. In a dense plasma, containing heavy ions, T_e is severely limited by radiation and temperatures in excess of 5 ev are unlikely. The actual value of T_e in a coaxial gun can be estimated as follows. The excitation coefficient S_j for an excitation energy of χ_j electron volts is given approximately by[6]

$$S_j = (6.0 \times 10^{-6}/\chi_j T_e^{1/2}) f \exp(-\chi_j/T_e) \text{ cm}^3 \text{ sec}^{-1} \tag{12}$$

where f is the oscillator strength and will be approximate to unity. By definition

$$\dot{n}_{j+1} = n_e n_j S_j \tag{13}$$

where n_e is the electron density, and n_j is the density of ions in the jth excited state. Hence P_e, the power going into excitation per unit volume, can be approximated as follows:

$$P_e = \sum_j \dot{n}_{j+1}\chi_j \approx n_e^2 S\chi \tag{14}$$

where S and χ refer to the strongest transition. Equation (14) determines T_e once n_e and P_e are known. The power P_e is unlikely to exceed the power per unit volume supplied to plasma motion, namely $En_e\bar{u}$ where E is the directed energy of an exhaust ion. This fact used with Eqs. (12) and (14) gives

$$T_e^{1/2} \exp(\chi/T_e) \approx 6.0 \times 10^{-6}\, n_e/E\bar{u} \tag{15}$$

Typical values, of interest to propulsion, for E and $\bar{u}$ are 100 ev and 5×10^6 cm sec^{-1}, respectively; substituting these values into Eq. (15) gives

$$T_e^{1/2} \exp(\chi/T_e) \approx 10^{-14}\, n_e \qquad (16)$$

The initial neutral atom density in a pulsed coaxial gun typically is 3×10^{15} cm^{-3} (i.e., a filling pressure of 40 mtorr); however, an electron density several times greater can be expected because of compression and multiple ionization. A reasonable estimate is 10^{16} cm^{-3}, in which case

$$T_e^{1/2} \exp(\chi/T_e) \approx 10^2 \qquad (17)$$

and T_e is determined in terms of the excitation potential χ.

Figure 2 shows $T_e^{1/2} \exp(\chi/T_e)$ plotted against T_e for different values of χ. With the exception of ions having one or two orbital electrons, χ is approximately 10 ev; near this value of excitation energy, T_e is limited to about 5 ev. It should be noted that T_e is determined primarily by the exponential term containing χ and is insensitive to the values of n_e, $\bar{u}$, and E.

The preceding argument has not mentioned what happens to the excitation energy. Either it will be radiated or it will constitute one step in the cascade process leading to ionization or radiation from a higher level. In neither case can the energy be used to produce useful momentum. (It is unlikely that an appreciable amount of excitation can be reclaimed by superelastic collisions.)

The energy in ion thermal motion may also be reclaimed by expansion; the likely magnitude of this effect is estimated next. If wall losses are ignored, then ion thermal energy is lost mainly by conduction to the electrons via Coulomb collisions; the electrons subsequently lose this energy by inelastic collisions with ions and neutrals at the rate given by Eq. (14). While the ions are much hotter than the electrons, T_i drops exponentially with an e-folding time given by[7]

$$\tau = 1.7 \times 10^7\, T_e^{3/2}\, A/(n_e Z^2) \text{ sec} \qquad (18)$$

where A is the atomic weight of the ions, and Z is the number of electronic charges on each ion. Because T_e is restricted to approximately 5 ev, the time τ given by Eq. (18) is short as compared with the acceleration time in typical coaxial guns, and consequently the plasma cannot store an appreciable amount of ion thermal energy; this point can be illustrated by substituting representative figures into Eq. (18). For $n_e = 10^{16}$ cm^{-3} and $A = 20$, the e-folding time for singly-ionized particles is 0.4 μsec and for doubly ionized particles is 0.1 μsec.

IV. Discussion

The obvious way to minimize the energy loss in a pulsed coaxial gun is to accelerate a discrete slug of gas into a vacuum. This solution has the merit of being simple and direct; its main drawback is that experiments have shown that the current sheet in a coaxial gun can become unstable unless it continually sweeps up gas.[8] Such instability may prove to be a limiting factor to an accelerator working in this fashion.

If an appreciable mass of gas is swept up by the current sheet, then for maximum efficiency large losses of thermal energy must be prevented, so that this energy can be reclaimed by expansion. One approach is to use a low-density plasma ($n_e \sim 10^{14}$ cm^{-3}) in which the energy loss rate could be small; however, it is unlikely that a pulsed gun would function properly in this density regime. Some improvement might be achieved by the correct choice of propellant. For instance, the first excitation potential of singly-ionized lithium is 62 v, and with it as propellant an electron temperature of 20–30 ev can be expected on the basis of Fig. 2. In practice, the electron temperature would probably be nearer 10–15 ev because, in this temperature regime, a prodigious amount of power can be radiated by small amounts of contaminant.[6] Although the electron temperature would certainly be higher with lithium than with a heavier propellant, the ion cooling rate would not be appreciably longer because of the low atomic weight. Alternatively, if a very heavy propellant is used, then the ions can retain their energy for a long time even in the presence of cold electrons; this last approach is attractive because the shock processes initially heat the ions; however excessive wall losses may result because of the large ion gyroradius.

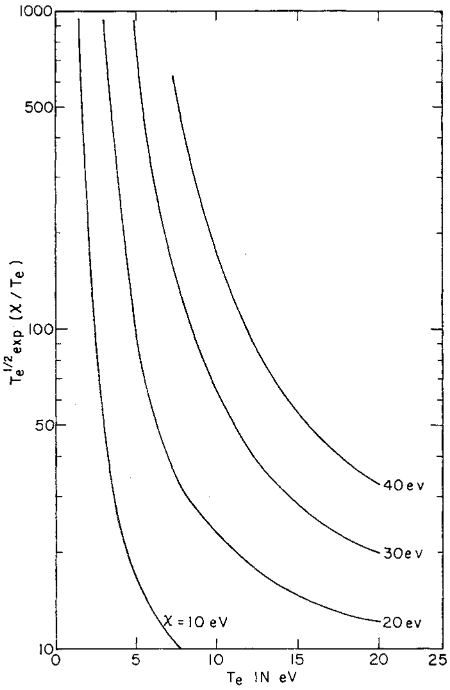

Fig. 2 $T_e^{1/2} \exp(\chi/T_e)$ plotted against T_e for various values of χ.

V. Conclusions

It is concluded that the thrustor efficiency in a pulsed coaxial gun is critically dependent upon the amount of neutral gas swept up by the plasma as it accelerates. For efficient operation, a large fraction of the energy should be supplied to the gun while the plasma is accelerating into a vacuum. This requirement arises because pulsed plasma accelerators normally operate in a density regime where thermal energy is lost by excitation and subsequent radiation or ionization before expansion can occur.

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